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Local Stability of Cournot Equilibrium as the Number of Firms Increases

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Abstract

In this paper, a Cournot oligopoly with isoelastic demand function and constant marginal cost is considered. The local stability conditions of the Cournot equilibrium are determined for four models with different decision mechanisms. In the first model, firms adjust their outputs using the best reply response with naive expectations. The second model is a generalization of the first one, where firms have adaptive expectations. Meanwhile, the third and fourth models adopt the bounded rationality and local monopolistic approximation, respectively. The results show that, in the case of identical firms, the Cournot equilibrium is always stable when the firms adopt the local monopolistic approximation mechanism.

Keywords: Cournot oligopoly, Isoelastic demand function, Local stability

1.0 Introduction

One of the key features which define a market structure is the number of firms in it. Monopoly and perfect competition are the two opposite ends of this structure. A monopoly is a market dominated by one firm while a perfectly competitive market has many firms such that no firm has any influence on the price of a product. Oligopoly is the intermediate structure between these two structures. It is a market dominated by few firms such that each firm's decision influences the other firms.

The concept of oligopoly was originated by A. Cournot in 1838 [1], who considered two firms competing by adjusting their outputs to maximize profits. Decision mechanisms play an important role in the output adjustment process. In literature, some common mechanisms include the naive expectation, adaptive expectation, bounded rationality, and local monopolistic approximation (LMA) [2-5].

The theory of perfect competition states that as the number of firms in a market increases, the equilibrium of the market becomes stable and the market becomes perfectly competitive. This theory is at odds with the results presented in a seminal paper by Theocharis [6], which shows that equilibrium in a linear Cournot oligopoly with naive expectation and discrete time scale becomes unstable when there are more than three firms. Naturally, this result has motivated several authors to extend the Theocharis problem in various directions.

The earliest extensions of the Theocharis problem was by Fisher [7] and McManus & Quandt [8], who showed that the equilibrium of a nonlinear Cournot oligopoly is always locally stable if the time scale is continuous instead of discrete. Ahmed & Agiza [9] and Agiza [10] showed that the result by Theocharis still holds in a nonlinear Cournot oligopoly with isoelastic demand function. However, by incorporating capacity limits into the Cournot model, destabilization of the equilibrium is avoided even for a large number of firms [11].

Another approach to the Theocharis problem is changing the decision mechanism in the adjustment process. By using adaptive instead of naive expectation in a linear Cournot oligopoly, the equilibrium remains stable when the number of firms increases, provided that the adaptive adjustment is small

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[12]. Tramontana *et al*. [13] analyzed a Cournot oligopoly with isoelastic demand and linear cost functions in the case of two, three, and four heterogeneous firms. Heterogeneity here refers to different decision mechanisms adopted by each firm. Their results show that when four firms differentially adopt a naive expectation, adaptive expectation, bounded rationality, or LMA, the equilibrium is still locally stable. More recently, Zhang and Gao [14] showed that the stability region of equilibrium in a Cournot oligopoly with quadratic cost function can be enlarged if the firms adopt LMA as their decision mechanism.

Collectively, these past studies suggested that the region of stability of Cournot equilibrium is influenced by the type of model and decision mechanism involved. Their results indicate that when the oligopoly model is nonlinear, the region of stability is slightly extended beyond three firms. However, by changing the decision mechanism from naive to adaptive expectation, subject to certain conditions on the coefficients in the model, the equilibrium can be stable for really large number of firms. In this paper, a similar Cournot oligopoly to that of Tramontana *et al*. [13] is analyzed for n homogenous firms for four cases that differ based on the decision mechanism adopted.

The paper is organized as follows. In Section 2, the Cournot oligopoly model and its equilibrium are derived. The local stability conditions of the equilibrium are determined for four Cournot oligopoly models in Sections 3 - 6, where the first, second, third, and fourth models adopted the naive expectation, adaptive expectation, bounded rationality, and local monopolistic approximation, respectively. Conclusions are presented in Section 7.

2.0 Cournot Oligopoly Model

Consider a market with n firms, each producing a single homogeneous product x_i for $i = 1, 2, \ldots, n.$ Assume the market is governed by the Cobb-Douglas utility function, so that its demand function is

$$X = \frac{1}{p}, \tag{1}$$

where $X = \sum_{i=1}^{n} x_i$ and p is the price of the product. In a Cournot oligopoly, the goal of each firm is to maximize its profit. The profit for each firm is the difference between total sales revenue and total cost, where total sales revenue is given by px_i. If we assume that the production cost per unit is c_i for each firm, then the total production cost for firm i is $c_i x_i$. So, the profit for firm i is

$$\pi_i = \frac{x_i}{X} - c_i x_i, \tag{2}$$

with marginal profit

$$\frac{\partial \pi_i}{\partial x_i} = \frac{X_i}{X^2} - c_i, \tag{3}$$

where $X_i = X - x_i$. By the first order condition, the profit maximizing output of firm i is

$$x_i = f_i(X_i) = \begin{cases} \sqrt{\dfrac{X_i}{c_i}} - X_i, & \text{if} \quad 0 < X_i < \dfrac{1}{c_i}, \\ 0, & \text{if} \quad \dfrac{1}{c_i} < X_i, \end{cases} \tag{4}$$

which is also called the best reply response or reaction function of firm i.

The Cournot equilibrium is the intersection point of the reaction functions of all firms. Rearranging equation (4) so that

$$x_i = X - c_i X^2, \tag{5}$$

and then taking the sum of x_i, for all $i = 1, 2, \ldots, n$, result in

$$CX^2 - (n-1)X = 0, \tag{6}$$

where $C=\sum_{i=1}^{n} c_i$, and the roots are $X=0$ and $X=(n-1)/C$. Let E be the Cournot equilibrium. Substituting these roots in (5) gives $E=(0,0,\dots,0)$ and

$$E=(e_1,e_2,\dots,e_n),\quad e_i=\frac{(n-1)(C+c_i-nc_i)}{C^2}, \tag{7}$$

where $e_i>0$, provided that $C>(n-1)c_i$. In the context of this paper, a zero output at the equilibrium point does not make sense, so the origin point is ignored. Note that this Cournot equilibrium has been derived previously by Matsumoto and Szidarovzsky [15].

3.0 Naive Expectation

Let $x_i(t+1)$ be output of firm i in period $t+1$. If firms are assumed to have perfect information of the demand function in a market, they adjust their output using the best reply response in (4), i.e.

$$x_i(t+1)=f_i\left(X_i^e(t+1)\right)=\begin{cases}\sqrt{\dfrac{X_i^e(t+1)}{c_i}}-X_i^e(t+1), & \text{if } \quad 0<X_i^e(t+1)<\dfrac{1}{c_i},\\[2ex] 0, & \text{if } \quad \dfrac{1}{c_i}<X_i^e(t+1),\end{cases} \tag{8}$$

where $X_i^e(t+1)$ is the expectation of firm i on the total output of the rest of the market in period $t+1$.

If firm i naively expects the rest of the market to produce the same amount as the last period, i.e. $X_i^e(t+1)=X_i(t)$, then (8) becomes an *n*-dimensional discrete dynamical system

$$x_i(t+1)=f_i\left(X_i(t)\right)=\begin{cases}\sqrt{\dfrac{X_i(t)}{c_i}}-X_i(t), & \text{if } \quad 0<X_i(t)<\dfrac{1}{c_i},\\[2ex] 0, & \text{if } \quad \dfrac{1}{c_i}\le X_i(t).\end{cases} \tag{9}$$

The Jacobian matrix of system (9), evaluated at the Cournot equilibrium in (7), is

$$J(E)=\begin{bmatrix}0 & k_1 & k_1 & k_1\\ k_2 & 0 & \cdots & k_2\\ \vdots & \vdots & \ddots & \vdots\\ k_n & \cdots & k_n & 0\end{bmatrix},\quad k_i=\frac{C}{2(n-1)c_i}-1. \tag{10}$$

Since the eigenvalues of this Jacobian matrix cannot be computed, the condition under which the Cournot equilibrium E in (7) is locally stable cannot be determined. However, by assuming $c_i=c_j=c$ for $i,j=1,2,\dots,n$, the Cournot equilibrium becomes

$$E_s=(e,e,\dots,e),\quad e=\frac{n-1}{n^2c}, \tag{11}$$

The Jacobian matrix in (10) becomes a symmetric matrix

$$J(E_s)=\begin{bmatrix}0 & k & k & k\\ k & 0 & \cdots & k\\ \vdots & \vdots & \ddots & \vdots\\ k & \cdots & k & 0\end{bmatrix},\quad k_i=\frac{2-n}{2(n-1)}, \tag{12}$$

with eigenvalues

$$\lambda_1 = \frac{2-n}{2}, \quad \lambda_{2,\ldots,n} = \frac{n-2}{2(n-1)}. \tag{13}$$

An equilibrium point of a system is locally stable if $|\lambda_i| < 1$ for $i = 1, 2, \ldots, n$. Based on Table 1, which summarizes the eigenvalues in (13) computed at $n = 2, 3, 4$ and 5 and the corresponding stabilities of E_s, the local stability condition for E_s in system (9) with common cost c is $2 \le n < 4$.

Table 1-Stability of E_s in system (9) with common c as n increases

n	λ_1	$\lambda_{2,\ldots,n}$	Stability of E_s
2	0	0	Super-stable
3	-0.5	0.25	Locally stable
4	-1	1/3	Marginally stable
5	-1.5	0.375	Unstable

4.0 Adaptive Expectation

In adaptive expectation, a firm does not reach the optimal output in every period. Instead, it applies some adjustments to the best reply response in (4). Mathematically, adaptive expectation is a general form of naive expectation given by

$$x_i(t+1) = (1-\alpha_i)x_i(t) + \alpha_i f_i(X_i(t)), \quad \alpha_i \in [0,1], \tag{14}$$

where α_i is the adaptive adjustment of firm i. The discrete dynamical system in (9) then becomes

$$x_i(t+1) = \begin{cases} (1-\alpha_i)x_i(t) + \alpha_i\left(\sqrt{\dfrac{X_i(t)}{c_i}} - X_i(t)\right), & \text{if } x_i(t) > \dfrac{\alpha_i}{1-\alpha_i}\left(X_i(t) - \sqrt{\dfrac{X_i(t)}{c_i}}\right), \\ 0, & \text{if } x_i(t) \le \dfrac{\alpha_i}{1-\alpha_i}\left(X_i(t) - \sqrt{\dfrac{X_i(t)}{c_i}}\right). \end{cases} \tag{15}$$

Note that at $\alpha_i = 1$, the adjustment process in (15) is reduced to the adjustment process in (9).
The Jacobian matrix of system (15), evaluated at the Cournot equilibrium in (7), is

$$J(E) = \begin{bmatrix} 1-\alpha_1 & \alpha_1 k_1 & \cdots & \alpha_1 k_1 \\ \alpha_2 k_2 & 1-\alpha_2 & \cdots & \alpha_2 k_2 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_n k_n & \cdots & \alpha_n k_n & 1-\alpha_n \end{bmatrix}. \tag{16}$$

Similar to the naive expectation model, the condition under which E is locally stable cannot be determined. However, by assuming $c_i = c_j = c$ and $\alpha_i = \alpha_j = \alpha$ for $i, j = 1, 2, \ldots, n$, the Jacobian matrix in (16) becomes

$$J(E_s) = \begin{bmatrix} 1-\alpha & \alpha k & \cdots & \alpha k \\ \alpha k & 1-\alpha & \cdots & \alpha k \\ \vdots & \vdots & \ddots & \vdots \\ \alpha k & \cdots & \alpha k & 1-\alpha \end{bmatrix}, \tag{17}$$

with eigenvalues

$$\lambda_1 = 1 - \frac{\alpha n}{2}, \quad \lambda_{2,\ldots,n} = 1 - \frac{\alpha n}{2(n-1)}, \tag{18}$$

where $n > 1$. The first eigenvalue is inside the unit circle if $0 < \alpha n < 4$ while the remaining $n-1$ eigenvalues are inside the unit circle if $0 < \alpha n < 4(n-1)$. Table 2 summarizes the stability conditions when the number of firms increases.

Table 2-Local stability conditions of E_s in system (15) with common c and α as n increases.

n	$0 < \alpha n < 4$	$0 < \alpha n < 4(n-1)$	Stability of E_s
2	$0 < \alpha < 2$	$0 < \alpha < 2$	Since $\alpha \in [0,1]$ (refer to equation (14)), then E_s is always locally stable since its local stability condition is $\alpha \in [0,2]$.
3	$0 < \alpha < 4/3$	$0 < \alpha < 8/3$	Since $\alpha \in [0,1]$ (refer to equation (14)), then E_s is always locally stable since its local stability condition is $\alpha \in [0,4/3]$.
4	$0 < \alpha < 1$	$0 < \alpha < 3$	Always locally stable since $\alpha \in [0,1]$
5	$0 < \alpha < 0.8$	$0 < \alpha < 3.2$	Locally stable if $0 < \alpha < 0.8$
6	$0 < \alpha < 2/3$	$0 < \alpha < 10/3$	Locally stable if $0 < \alpha < 2/3$

Based on Table-2, since $\alpha \in [0,1]$, E_s is always locally stable in system (15) with common cost c and common adaptive adjustment α when $n = 2,3,4,$ while for $n \geq 5$, it is locally if $0 < \alpha < 4/n$. In Figure-1, the region of stability for E_s as the number of firms increases shows that $\alpha \to 0$ as $n \to \infty$. This implies that when the number of firms increases, the market or system is stable if firms employ small adaptive adjustment.

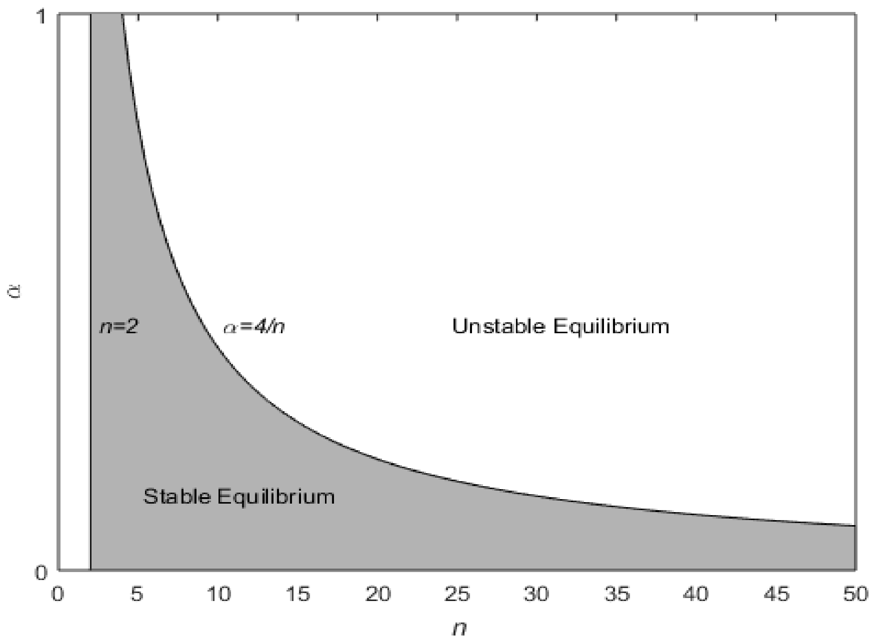

Figure 1-Stability region of E_s in system (15) with common c and a Î $[0,1]$ as n increases.

5.0 Bounded Rationality

If firms are assumed to have limited information of the market, they can adjust their output by using their marginal profits instead of using the best reply response. Consider a market where all firms update their outputs in period $t+1$ by increasing (decreasing) its output if the marginal profit is positive (negative). This adjustment process, called bounded rationality adjustment, is given by

$$x_i\left(t+1\right)=x_i(t)+\beta_i\frac{\partial\pi_i\left(X_i(t),X(t)\right)}{\partial x_i(t)},\quad \beta_i>0, \tag{19}$$

where β_i is the speed of adjustment for firm i. Therefore, the discrete dynamical system of a market where n firms adopt bounded rationalities in adjusting their outputs is

$$x_i\left(t+1\right)=\begin{cases} x_i(t)+\beta_i\left(\dfrac{X_i\left(t\right)}{\left(x_i(t)+X_i(t)\right)^2}-c_i\right), & \text{if}\quad X_i>\dfrac{x_i(t)\left(c_i\beta_i-x_i(t)\right)}{x_i(t)+\beta_i\left(1-c_i\right)},\\[2ex] 0, & \text{if}\quad X_i\le\dfrac{x_i(t)\left(c_i\beta_i-x_i(t)\right)}{x_i(t)+\beta_i\left(1-c_i\right)}. \end{cases} \tag{20}$$

The Jacobian matrix of system (20), evaluated at the Cournot equilibrium in (7), is

$$J\left(E\right)=\begin{bmatrix} 1-u_1 & v_1-u_1 & \cdots & v_1-u_1\\ v_2-u_2 & 1-u_2 & \cdots & v_2-u_2\\ \vdots & \vdots & \ddots & \vdots\\ v_n-u_n & \cdots & v_n-u_n & 1-u_n \end{bmatrix},\quad u_i=\frac{2c_i\beta_iC}{n-1},\quad v_i=\frac{\beta_iC^2}{\left(n-1\right)^2}. \tag{21}$$

By assuming $c_i=c_j=c$ and $\beta_i=\beta_j=\beta$ for $i,j=1,2,\ldots,n$, the Jacobian matrix becomes

$$J\left(E_s\right)=\begin{bmatrix} 1-u & v-u & \cdots & v-u\\ v-u & 1-u & \cdots & v-u\\ \vdots & \vdots & \ddots & \vdots\\ v-u & \cdots & v-u & 1-u \end{bmatrix},\quad u=\frac{2nc^2\beta}{n-1},\quad v=\frac{\beta n^2c^2}{\left(n-1\right)^2}, \tag{22}$$

with eigenvalues

$$\lambda_1=1-\frac{\beta n^2c^2}{n-1},\quad \lambda_{2,\ldots,n}=1-\frac{\beta n^2c^2}{\left(n-1\right)^2}, \tag{23}$$

where $n>1$. Since $\left|\lambda_i\right|<1$ for $i=1,2,\mathrm{K},n$, then the stability conditions from the eigenvalues in (23) are $0<\beta c^2<\dfrac{2\left(n-1\right)}{n^2}$ and $0<\beta c^2<\dfrac{2\left(n-1\right)^2}{n^2}$. Table 3 summarizes these stability conditions as n increases. Based on Table 3, E_s is locally stable in system (20) with common cost c and common speed of adjustment b if

$$0<\beta c^2<\frac{2\left(n-1\right)}{n^2}. \tag{24}$$

Table 3-Local stability conditions of E_s in system (20) with common β and c as n increases.

n	$0<\beta c^2<\dfrac{2\left(n-1\right)}{n^2}$	$0<\beta c^2<\dfrac{2\left(n-1\right)^2}{n^2}$	Stability of E_s
2	$0<\beta c^2<1/2$	$0<\beta c^2<1/2$	Locally stable if $0<\beta c^2<1/2$

3	$0<\beta c^2<4/9$	$0<\beta c^2<8/9$	Locally stable if $0<\beta c^2<4/9$
4	$0<\beta c^2<3/8$	$0<\beta c^2<9/8$	Locally stable if $0<\beta c^2<3/8$
5	$0<\beta c^2<8/25$	$0<\beta c^2<32/25$	Locally stable if $0<\beta c^2<8/25$
6	$0<\beta c^2<5/18$	$0<\beta c^2<50/36$	Locally stable if $0<\beta c^2<5/18$

Figure-2 shows that the region of stability for E_s in system (20) becomes smaller as the number of firms increases. Note that the largest value for $b c^2$ is 0.5. Therefore, the stability condition for E_s in (24) should be rewritten as

$$\frac{2(n-1)}{n^2} < b c^2 < 0.5, \quad n > 2. \tag{25}$$

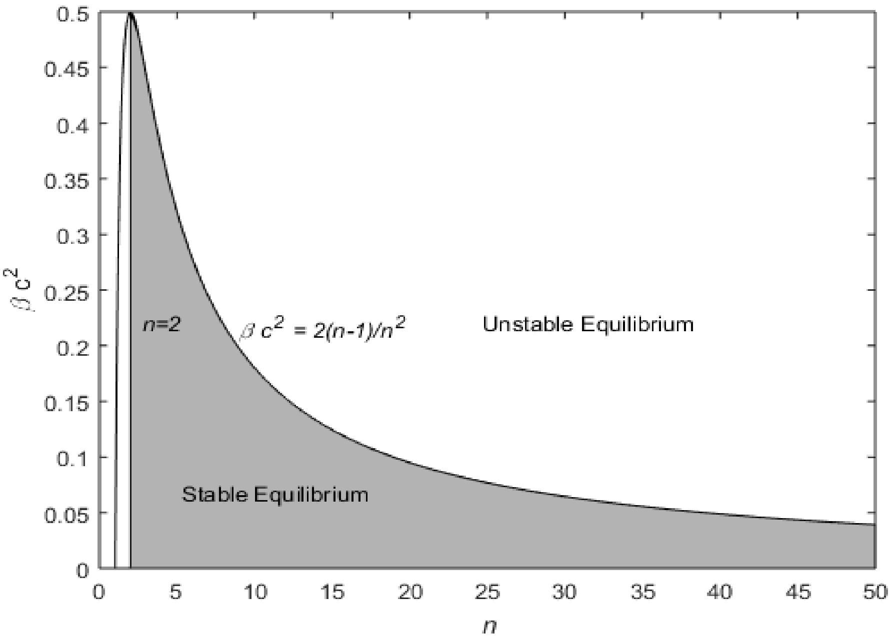

Figure 2-Stability region of E_s in system (20) with common c and β as n increases.

6.0 Local Monopolistic Approximation

If firms are assumed to only know a point on the demand function, they can attempt to obtain a linear approximation of the demand function using their local knowledge. In particular, firm *i* can compute the effects of small quantity variations x_i on the price *p* in period *t*. This variation can be represented as a partial derivative of the price with respect to quantity in period *t*. The expected price of firm i in period $t+1$ can then be computed using the following rule of estimation:

$$p_i(t+1) = p(t) + \frac{\partial p(t)}{\partial x_i(t)}\left(x_i(t+1) - x_i(t)\right). \tag{26}$$

where $p(t) = 1/X(t)$ as defined in (1). This type of adjustment is called local monopolistic approximation (LMA). Using this expected price, the profit of firm i in period $t+1$ is $\pi_i(t+1) = p(t+1)x_i(t+1) - c_i x_i(t+1)$, i.e.

$$\pi_i(t+1)=\left(\frac{1}{X(t)}+\frac{x_i(t)-x_i(t+1)}{X^2(t)}\right)x_i(t+1)-c_i x_i(t+1). \tag{27}$$

By the first order condition,

$$\frac{\partial \pi_i(t+1)}{\partial x_i(t+1)}=\frac{1}{x_i(t)+X_i(t)}+\frac{x_i(t)}{\left(x_i(t)+X_i(t)\right)^2}-\frac{2x_i(t+1)}{\left(x_i(t)+X_i(t)\right)^2}-c_i, \tag{28}$$

thus yielding the reaction function

$$x_i(t+1)=x_i(t)+\frac{X_i(t)}{2}-c_i\frac{\left(x_i(t)+X_i(t)\right)^2}{2}. \tag{29}$$

Therefore, when n firms adopt LMA as their decision mechanism, the discrete dynamical system of the market becomes

$$x_i(t+1)=\begin{cases} x_i(t)+\dfrac{X_i(t)}{2}-c_i\dfrac{\left(x_i(t)+X_i(t)\right)^2}{2}, & \text{if} \quad x_i(t)>c_iX^2(t)-X(t), \\ 0, & \text{if} \quad x_i(t)\le c_iX^2(t)-X(t). \end{cases} \tag{30}$$

The Jacobian matrix of system (30), evaluated at E in (7), is

$$J(E)=\begin{bmatrix} m_1 & m_1/2 & \cdots & m_1/2 \\ m_2/2 & m_2 & \cdots & m_2/2 \\ \vdots & \vdots & \ddots & \vdots \\ m_n/2 & \cdots & m_n/2 & m_n \end{bmatrix}, \quad m_i=\frac{C-(n-1)c_i}{C}. \tag{31}$$

By assuming $c_i=c_j=c$ for $i,j=1,2,\dots,n$, $J(E)$ becomes a symmetric matrix

$$J(E_s)=\begin{bmatrix} 1/n & 1/2n & \cdots & 1/2n \\ 1/2n & 1/n & \cdots & 1/2n \\ \vdots & \vdots & \ddots & \vdots \\ 1/2n & \cdots & 1/2n & 1/n \end{bmatrix}, \tag{32}$$

with eigenvalues

$$\lambda_1=\frac{n+1}{2n}, \quad \lambda_{2,\dots,n}=\frac{1}{2n}. \tag{33}$$

Since $|\lambda_1|<1$ and $|\lambda_{2,\dots,n}|<1$ if $n>1$ and $n>1/2$, respectively, then the local stability condition for E_s in system (30) with common cost c is $n>1$.

7.0 Conclusions

In this paper, the local stability conditions of a Cournot equilibrium in four different models was derived for n identical firms. In the first model, when n firms adopt naive expectation as their decision mechanisms, the equilibrium becomes unstable when $n>4$. Meanwhile, in the second, third, and fourth models, firms abandon this unrealistic expectation and adopt other decision mechanisms. In the second and third models, by adopting adaptive expectation and bounded rationality, respectively, the equilibrium can remain stable for really large number of firms, provided that the production cost and adjustment value are very small. In the last model, by adopting local monopolistic approximation (LMA), the equilibrium is always locally stable for any number of firms, with the exception of $n=1$. In general, these results indicate that firms are better off in adopting decision mechanisms which reflects realistic situation of the decision-making process in a competitive market. By adopting LMA, theoretically, a homogenous market is stable for any number of firms apart from a monopoly.

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