Topologized Soft Fundamental Group of Soft Topological Group

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Abstract
In this paper, we show that each soft topological group is a strong small soft loop transfer space at the identity element. This indicates that the soft quasitopological fundamental group of a soft connected and locally soft path connected space, is a soft topological group.

Keywords: fundamental group, locally path connected space, soft topological group, quasitopological fundamental group.

Introduction
The fundamental group awarded with the quotient topology is tempted by the natural surjective map
$$q : \Omega(X,x_0) \rightarrow \pi_1(X,x_0),$$
where $\Omega(X,x_0)$ is the loop space of $(X,x_0)$ with compact-open topology, signified by $\pi_1^{top}(X,x_0)$ and becomes a quasitopo-logical group [1,2]. Torabi et al. [3] showed that the quasitopological fundamental group of a connected locally path connected, semi locally small generated space, is a topological group. Spanier [4] presented a different topology on the fundamental group which was called the whisker topology by Brodskiy et al. [5] and signified by $\pi_1^{wh}(X,x_0)$.

Hamed Torabi [6] presented that the quasitopological fundamental group of a connected locally path connected topological group is a topological group.

In this paper we will demonstrate that the soft quasitopological fundamental group of a soft connected locally soft path connected topological group is a soft topological group.

Definition 1.1:
Let $(X,\mathcal{T})$ is a soft topology and let $V$ is an equivalent relationship on $X$ when $q(x) = [x]$ then $X/V$ is all soft equivalent classes, when $q: X \rightarrow X/V$ is a soft quotient function then $\overline{\mathcal{T}} = \{A \subseteq X/V : q^{-1}(A) \in \mathcal{T}\}$ is soft quotient topology and $(X/V,\overline{\mathcal{T}})$ is termed soft quotient topological space.

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the soft fundamental group $\pi_1(X, x_0)$ with the soft quotient topology $X/\mathcal{V}$ is signified by $\pi_1^{\text{sqtop}}(X, x_0)$.

**Theorem 1.2.**

Let $(X, x)$ be a soft locally path connected soft pointed space and $\mathcal{V} = \{V_\alpha \mid \alpha \in \mathcal{P}(X, x)\}$ be a soft path open cover of $(X, x)$. Then $\tilde{\pi}(\mathcal{V}, x)$ is a soft open subgroup of $\pi_1^{\text{sqtop}}(X, x_0)$.

**Definition 1.3:**

Let $\tilde{X}$ is the soft space of soft homotopy classes of based soft paths in $X$. For any soft pointed topological space $(X, x_0)$ the soft whisker topology on $\tilde{X}$ is defined by the soft basis $B(\{\alpha\}, U) = \{[\alpha \ast \beta] \mid \beta \in \mathcal{U}\}$, where $\alpha$ is a soft path in $X$ from $x_0$ to $x_1$, $U$ is a soft neighborhood of $x_1$ in $X$, and $\beta$ is a soft path in $U$ originating at $x_1$, the soft path $\beta$ is said to be a $U$-soft whisker. We represent $\tilde{X}$ with the soft whisker topology by $\tilde{X}_{\text{swh}}$.

The soft fundamental group $\pi_1(X, x_0)$ with the soft subspace topology congenital from $\tilde{X}_{\text{swh}}$ is signified by $\pi_1^{\text{swh}}(X, x_0)$.

**Definition 1.4:**

A soft topological space $X$ is called a small soft loop transfer (SSLT for short) space at $x$ if for each soft path $\alpha$ in $X$ with $\alpha(0) = x_0$ and for each soft neighborhood $U$ of $x_0$ there is a soft neighborhood $V$ of $\alpha(1) = x$ such that for each soft loop $\beta$ in $V$ based at $x$ there is a soft loop $\gamma$ in $U$ based at $x_0$ which is soft homotopic to $\alpha \ast \beta \ast \alpha^{-1}$ corresponding to $I$. The soft space $X$ is called an SSLT space if $X$ is SSLT at $x_0$ for each $x_0 \in X$.

**Definition 1.5:**

Let $(X, x_0)$ be a soft locally path connected space and let $\mathcal{V} = \{V_\alpha \mid \alpha \in \mathcal{P}(X, x_0)\}$ be a soft path open cover of $X$ by the soft neighborhoods $V_\alpha$ excluding $\alpha(1)$. Define $\tilde{\pi}(\mathcal{V}, x_0)$ as the subgroup of $\pi_1(X, x_0)$ involving of the soft homotopy classes of soft loops that can be represented by a product of the following form

$$\prod_{j=1}^{n} \alpha_j \beta_j \alpha_j^{-1}$$

where $\alpha_j$ are arbitrary soft path starting at $x_0$ and each $\beta_j$ is a soft loop inside the soft open set $V_{\alpha_j}$, for all $j \in \{1, 2, \ldots, n\}$. We call $\tilde{\pi}(\mathcal{V}, x_0)$ the soft path Spanier group of $\pi_1(X, x_0)$ with admiration to $\mathcal{V}$.

**Proposition 1.6:**

Let $X$ is a soft topological space, then we say that $X$ is SSLT at $x_0$ if and only if for each soft open neighborhood $U \subseteq X$ enclosing $x_0$ is found a soft path open cover $\mathcal{V}$ of $X$ at $x$ such that $\tilde{\pi}(\mathcal{V}, x_0) \leq i_\ast \pi_1(U, x_0)$.

**Proof:**

Let $U$ be a soft open neighborhood of $X$. As $X$ is SSLT at $x_0$, for each soft path $\alpha$ from $x_0$ to $\alpha(1)$ there is a soft open neighborhood $V_{\alpha}$ of $\alpha(1)$ such that for each soft loop $\beta$ in $V_{\alpha}$ based at $\alpha(1)$ we have $[\alpha \ast \beta \ast \alpha^{-1}] \in i_\ast \pi_1(U, x_0)$, where $i_\ast : \pi_1(U, x_0) \rightarrow \pi_1(X, x_0)$ is the homomorphism convinced by the soft inclusion map $i : U \rightarrow X$. Consider $\mathcal{V} = \{V_\alpha \mid \alpha \in \mathcal{P}(X, x_0)\}$. Hence each generator of $\tilde{\pi}(\mathcal{V}, x_0)$ goes to $i_\ast \pi_1(U, x_0)$ which indicates that $\tilde{\pi}(\mathcal{V}, x_0) \leq i_\ast \pi_1(U, x_0)$.

On the other hand, let $\alpha$ be a soft path from $x_0$ to $\alpha(1)$ and $U$ be a soft open neighborhood enclosing $x_0$. By the definition of the soft path Spanier group, there is a $V_{\alpha} \in \mathcal{V}$ such that $[\alpha \ast \beta \ast \alpha^{-1}] \in \tilde{\pi}(\mathcal{V}, x_0)$ for each soft loop $\beta$ in $V_{\alpha}$ based at $\alpha(1)$. Thus, by assumption, $[\alpha \ast \beta \ast \alpha^{-1}] \in i_\ast \pi_1(U, x_0)$ which indicates that $\alpha$ is an SSLT path. Therefore $X$ is an SSLT space at $x_0$.

**Corollary 1.7:**

A soft topological space $X$ is SSLT at $x_0$ if and only if for each soft open neighborhood $U \subseteq X$ enclosing $x_0$, $i_\ast \pi_1(U, x_0)$ is an open subgroup of $\pi_1^{\text{sqtop}}(X, x_0)$. 

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Theorem 1.8:

Let $X$ be a soft connected locally path soft connected space, then $X$ is SSLT at $x_0$ if and only if $\pi^\text{swth}_1(X, x_0) = \pi^\text{sttop}_1(X, x_0)$.

Proof:

Let $X$ be SSLT at $x_0$. It is ample to expression that $\pi^\text{swth}_1(X, x_0)$ is bristlier than $\pi^\text{sttop}_1(X, x_0)$. Let the assortment $\{[\alpha] : i_n^1(U, x_0) \mid [\alpha] \in \pi_1(X, x_0)\}$ custom a basis for the soft whisker topology on $\pi_1(X, x_0)$. Thus, it be enough to verify that $[\alpha] \cdot i_n^1(U, x_0)$ is a soft open subset of $\pi^\text{sttop}_1(X, x_0)$, where $U$ is a soft open neighborhood of $x_0$. Using Proposition 1.6, there is a soft path open cover $V$ of $X$ such that $\hat{U}(V, x_0) \leq i_n^1(U, x_0)$. Since $\hat{U}(V, x_0)$ is soft open in $\pi^\text{sttop}_1(X, x_0)$ (Theorem 1.2) and $\pi^\text{sttop}_1(X, x_0)$ is a soft quasitopological group, we imply that $[\alpha] \cdot i_n^1(U, x_0)$ is a soft open subset of $\pi^\text{sttop}_1(X, x_0)$.

On the other hand, assume $\pi^\text{swth}_1(X, x_0) = \pi^\text{sttop}_1(X, x_0)$. The subset $i_n^1(U, x_0)$ is a soft open basis in $\pi^\text{swth}_1(X, x_0)$. Then the subset $i_n^1(U, x_0)$ is soft open in $\pi^\text{sttop}_1(X, x_0)$. Therefore Corollary 2.4 indicates that $X$ is SSLT at $x_0$.

Corollary 1.9:

For a soft connected and soft locally path soft connected space $X$, if $X$ is SSLT at $x_0$, then $\pi^\text{sttop}_1(X, x_0)$ and $\pi^\text{swth}_1(X, x_0)$ are soft topological groups.

Note 1.10: [7]

if $\{f_\alpha : (X, T, E) \to (Y, S, F, E)\}$ is a family of soft continuous functions, then the soft function $\prod_{\alpha \in S} \times f_\alpha : \prod_{\alpha \in S} X_{\alpha} \times T, E \to \prod_{\alpha \in S} Y_{\alpha}, F, E$ is soft continuous.

2. Topologized soft fundamental group of soft topological group

Let $G$ be a soft topological group and $\alpha$ be a soft path in $G$, then we denote the soft homotopy class $\alpha$ by $[\alpha]$ and the inverse of $\alpha$ by $\bar{\alpha}$ where $\bar{\alpha} : I \to G$ by $\bar{\alpha}(t) = \alpha(1-t)$. Also we define $\alpha^{-1} : I \to G$ by $\alpha^{-1}(t) = (\alpha(t))^{-1}$ and denote the constant soft path $\alpha : I \to G$ at $\alpha \in G$ by $C_\alpha$.

Definition 2.1:

Let $G$ be a soft topological group with the multiplication soft function $m : G \times G \to G$, given by $(x, y) \to xy$. Let $\alpha, \beta$ be two soft paths in $G$. We define the soft path $\alpha \cdot \beta : I \to G$ by $\alpha \cdot \beta(t) = m(\alpha(t), \beta(t))$. Since the multiplication soft function and $\alpha, \beta$ are soft continuous $\alpha \cdot \beta : I \to G$ is soft continuous (by Note 1.10).

Lemma 2.2:

If $G$ is a soft topological group and $\lambda, \gamma$ be two soft loops in $G$ based at $a \in G$ and $b \in G$ respectively, then $[\lambda, \gamma] = [\lambda^a] \cdot [\gamma^b]$. In particular, if $\lambda, \gamma$ be two soft loops in $G$ based at the soft identity element $e_G$, then $[\lambda, \gamma] = [\lambda][\gamma]$.

Proof:

Consider the soft continuous multiplication function $m : G \times G \to G$, given by $(x, y) \to xy$. Let $\theta : \pi_1(G, a) \times \pi_1(G, b) \to \pi_1(G \times G, (a, b))$ be the soft isomorphism defined by $([\lambda], [\gamma]) \to ([\lambda], [\gamma])$. Since $m_\theta : \pi_1(G, a) \times \pi_1(G, b) \to \pi_1(G, ab)$ is a soft homomorphism and $([\lambda], [\gamma]) = ([\lambda], [\gamma])$, we have $m_\theta([\lambda], [\gamma]) = m_\theta([\lambda], [\gamma]) = m_\theta([\lambda], [\gamma])$. On the other hand $m_\theta([\lambda], [\gamma]) = [\lambda, \gamma]$, which indicates that $[\lambda^b][\gamma^c] = [\lambda, \gamma]$.

Definition 2.3:

A soft topological space $X$ is said to be a strong small soft loop transfer (strong SSLT) space at $x_0$ if for each $x \in X$ and for each soft neighborhood $U$ of $x_0$ there is a soft neighborhood $V$ of $x$ such that for each soft path $\alpha$ in $X$ with $\alpha(0) = x_0, \alpha(1) = x$ and for each soft loop $\beta$ in $V$ based at $x$.
there is a soft loop $y$ in $U$ based at $x_0$ which is soft homotopic to $\alpha * \beta * \bar{\alpha}$ relative to $I$. The soft space $X$ is said to be a strong SSLT space if $X$ is strong SSLT at $x_0$ for each $x_0 \in X$.

**Theorem 2.4:**

A soft topological group $G$ is a strong SSLT space at the identity element $e_G$.

**Proof:**

Let $U$ be a soft neighborhood of $e_G$ in $G$ and $x \in G$. We show that for each soft loop $\beta$ based at $x$ in the soft neighborhood $xU = \{ xu | u \in U \}$ of $x$ and each soft path $\alpha$ in $G$ with $\alpha(0) = e_G$, $\alpha(1) = x$, there is a soft loop $\tilde{\alpha}$ in $U$ based at $e_G$ which is soft homotopic to $\alpha * \beta * \bar{\alpha}$ relative to $I$. For this let $\lambda$ be a soft loop in $G$ based at $e_G$ such that

$$
\begin{cases}
\lambda(t) = \alpha(3t) & 0 \leq t \leq 1/3 \\
\alpha(3t - 2) & 2/3 \leq t \leq 1
\end{cases}
$$

Also let $\gamma$ be a soft loop in $G$ based at $e_G$ such that

$$
\begin{cases}
\gamma(t) = e_G & 0 \leq t \leq 1/3 \\
\gamma(t) = e_G & 2/3 \leq t \leq 1
\end{cases}
$$

Therefore by Lemma 2.2 we have $[\lambda][\gamma] = [\lambda, \gamma]$. Note that

$$
(\lambda, \gamma)(t) = \begin{cases}
\alpha(3t) & 0 \leq t \leq 1/3 \\
\beta(3t - 1) & 1/3 \leq t \leq 2/3 \\
\alpha(3t - 2) & 2/3 \leq t \leq 1
\end{cases}
$$

If $\xi$ is a soft loop in $U$ based at $e_G$ such that for each $t \in I$, $\gamma(t) = x^{-1}\beta(t)$ then we have $[\xi] = [\alpha * \bar{\alpha}][\xi] = [\lambda][\gamma] = [\lambda, \gamma] = [\alpha * \beta * \bar{\alpha}]$.

Hence $G$ is a strong SSLT space at $e_G$.

**Corollary 2.5:**

Let $G$ be a soft topological group. Then $G$ is an SSLT space at the identity element $e_G$.

**Corollary 2.6:**

Let $G$ be a soft connected and soft locally soft path connected soft topological group, then $\pi^{\text{soft}}_1(G, e_G) = \pi_{1, \text{swh}}(G, e_G)$ is a soft topological group.

**Proof:**

By corollary 2.5, $G$ is an SSLT space at $e_G$. Therefore $\pi^{\text{soft}}_1(G, e_G) = \pi_{1, \text{swh}}(G, e_G)$ by Theorem 1.8. Hence $\pi^{\text{soft}}_1(G, e_G)$ and $\pi_{1, \text{swh}}(G, e_G)$ are soft topological group by Corollary 1.9.

**Proposition 2.7:**

Let $G$ connected and locally soft path connected soft topological group and $H \leq \pi_1(G, e_G)$. Then the next statements are equivalent.

(i) $H$ is a soft open subgroup of $\pi^{\text{soft}}_1(G, e_G)$.

(ii) $H$ is a soft open subgroup of $\pi_{1, \text{swh}}(G, e_G)$.

(iii) There is a soft neighborhood $U$ of $e_G$ s.t. $i * \pi_1(U, e_G) \leq H$.

**Proof.** (i) $\iff$ (ii) deduce from Corollary 2.6.

(ii) $\implies$ (iii) : Let $H$ be a soft open subgroup of $\pi_{1, \text{swh}}(G, e_G)$. Since $i * \pi_1(V, e_G)$ is a soft open basis in $\pi_{1, \text{swh}}(G, e_G)$, then there is a soft neighborhood $U$ of $e_G$ s.t. $i * \pi_1(U, e_G) \leq H$.

(iii) $\implies$ (ii) : Let there is a soft neighborhood $U$ of $e_G$ s.t. $i * \pi_1(U, e_G) \leq H$. Since $i * \pi_1(U, e_G) \leq H$ is a soft open set in $\pi_{1, \text{swh}}(G, e_G)$ and $i * \pi_1(U, e_G) \leq H$ and $\pi_{1, \text{swh}}(G, e_G)$ is a soft topological group, Hence $H$ is a soft open subgroup of $\pi_{1, \text{swh}}(G, e_G)$.

**Definition 2.8:**

Let $H \leq \pi_1(X, x_0)$. A soft topological space $X$ is called an $H$-small soft loop transfer ($H$-SSLT) space at $x_0$ if for each soft path $\alpha$ in $X$ with $\alpha(0) = x_0$ and for each soft neighborhood $U$ of $x_0$ there is a soft neighborhood $V$ of $\alpha(1) = x$ such that for each soft loop $\beta$ in $V$ based at $x$ there is a soft loop $\gamma$ in $U$ based at $x_0$ such that $[\alpha * \beta * \bar{\alpha} * \bar{\gamma}] \in H$. 

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It is simple to see that each SSLT space at $x$ is an $H$-SSLT space at $x$, for any soft subgroup $H$ of $\pi_1(X,x_0)$, so each soft topological group $G$ is a $H$-SSLT space at $e_G$, for any soft subgroup $H$ of $\pi_1(G,e)$.

Theorem 2.9:

Let $H \leq \pi_1(X,x_0)$ and $X$ be a $H$-SSLT at $x_0$. So $X$ is soft homotopically path Hausdorff for $H$ iff $X$ is soft homotopically Hausdorff for $H$.

Lemma 2.10:

Let $C$ be a subset of $\pi_1(X,x_0)$ and $C \neq \pi_1(X,x_0)$. W say that $X$ is soft homotopically path-Hausdorff for $C$ if $C$ is closed in $\pi_1^{\text{qtop}}(X,x_0)$, and we say that $C$ is closed in $\pi_1^\text{soft}(X,x_0)$, if $X$ is soft homotopically path-Hausdorff for $C$ and soft locally soft path connected.

Proposition 2.11:

Let $G$ be a soft connected and soft locally soft path connected soft topological group and $H \leq \pi_1(G,e)$. So the next statements are equivalent.

(i) $H$ is a soft closed subgroup of $\pi_1^{\text{qtop}}(G,e)$.

(ii) $H$ is a soft closed subgroup of $\pi_1^{\text{soft}}(G,e)$.

(iii) $G$ is soft homotopically Hausdorff for $H$.

(iv) $G$ is soft homotopically soft path Hausdorff for $H$.

Proof:

(i) $\iff$ (ii) deduce from corollary 2.6.

(iii) $\iff$ (iv) deduce from Theorem 2.9, since $G$ is an $H$-SSLT at $e_G$ and $\pi_1(G,e_G)$ is abelian, so $H$ is a soft normal subgroup of $\pi_1(G,e_G)$.

(iv) $\iff$ (i) deduce from Lemma 2.10.

Corollary 2.12:

A soft connected locally soft path connected soft topological group $G$ is a soft homotopically Hausdorff if and only if $\pi_1^{\text{qtop}}(G,e_G)$ is a soft Hausdorff space.

Proof:

Assume that $G$ is a soft homotopically Hausdorff. So $G$ is a soft homotopically Hausdorff relative to the soft trivial subgroup $H = \{1\}$. Hence by Proposition 2.11 $\{e_G\}$ is closed in $\pi_1^{\text{qtop}}(G,e_G)$. Therefore for each $g \in G$, $\{g\}$ is closed in $\pi_1^{\text{qtop}}(G,e_G)$ since $\pi_1^{\text{qtop}}(G,e_G)$ is a soft quasitopological group. Hence $\pi_1^{\text{qtop}}(G,e_G)$ is $\mathcal{T}_G$, which indicates that it is a soft Hausdorff space since $\pi_1^{\text{qtop}}(G,e_G)$ is a soft topological group. The converse is trivial.

Theorem 2.13:

A soft topological group $G$ is a strong SSLT space if $G$ is an abelian group or a soft path connected space.

Proof:

Let $G$ be an abelian soft topological group and $a \in G$. We show that $G$ is a strong SSLT space at $a$. For this let $U$ be a soft neighborhood of $a$ in $G$ and $b \in G$. We show that for each soft loop $\beta$ based at $b$ in the soft neighborhood $bU = \{ba^{-1}u | u \in U\}$ of $b$ and each soft path $\alpha$ in $G$ with $\alpha(0) = a, \alpha(1) = b$, there is a soft loop $\gamma$ in $U$ based at $a$ which is soft homotopic to $\alpha * \beta * a$ relative to $I$. Let $\lambda$ be a soft loop in $G$ based at a such that

$$\lambda(t) = \begin{cases} 
\alpha(3t) & 0 \leq t \leq 1/3 \\
b & 1/3 \leq t \leq 2/3 \\
\alpha(3t-2) & 2/3 \leq t \leq 1
\end{cases}$$

Also let $\gamma$ be a soft loop in $G$ based at a such that

$$\gamma(t) = \begin{cases} 
e_G & 0 \leq t \leq 1/3 \\
ab^{-1}\beta(3t-1) & 1/3 \leq t \leq 2/3 \\
e_G & 2/3 \leq t \leq 1
\end{cases}$$

Therefore by Lemma 2.2 we have $[\lambda]^a = [\alpha, \gamma]$. Note that...
Since $G$ is abelian, hence $bab^{-1} = a, a^a = a$ and $\alpha a = \alpha a$. Therefore
\[
\lambda \gamma = ([a \lambda] * ([a \gamma]) = [a \lambda] [a \gamma] = [a \gamma] = [a (\alpha \beta * \alpha)].
\]
Since $G$ is abelian, so $\lambda \gamma = [a \lambda] = [a \beta]$. Hence
\[
\lambda \gamma = ([a \lambda] * ([a \gamma]) = [a \lambda] [a \gamma] = [a \gamma] = [a (\alpha \beta * \alpha)].
\]
Therefore $\lambda \gamma = ([a \lambda] * ([a \gamma]) = [a \lambda] [a \gamma] = [a \gamma] = [a (\alpha \beta * \alpha)]$.

Now let $G$ be a soft path connected soft topological group and $a \in G$. We show that $G$ is a strong SSLT space at $a$. For this let $U$ be a soft neighborhood of $a$ in $G$ and $b \in G$. We show that for each soft loop $\beta$ based at $b$ in the soft neighborhood $ba^{-1}U = \{ba^{-1}u | u \in U\}$ of $b$ and each soft path $\alpha$ in $G$ with $\alpha(0) = a, \alpha(1) = b$, there is a soft loop $f$ in $U$ based at $a$ which is soft homotopic to $\alpha \beta * \alpha$ relative to $I$. Since $G$ is soft path connected so there is a soft path $\gamma$ in $G$ from $e_G$ to $a$. By proof of Theorem 2.4, we have
\[
\delta = [\delta] = [\alpha * \beta * \alpha * \beta] = [(g \alpha \beta * (g \alpha \beta)] = [(g \alpha \beta * (g \alpha \beta)] [b^{-1} \gamma] = [C_{b^{-1}}] [b^{-1} \beta]
\]
Also
\[
\gamma = (g \alpha \beta * (g \alpha \beta)] = [(g \alpha \beta * (g \alpha \beta)] [b^{-1} \gamma] = [C_{b^{-1}}] [b^{-1} \beta] = [b^{-1} \gamma].
\]
Therefore $[g \alpha \beta * (g \alpha \beta)] = [g \alpha \beta * (g \alpha \beta)] [b^{-1} \gamma]$, which indicates that $[\alpha \beta * \alpha] = [ab^{-1} \beta]$.

If $f = b^{-1} \beta$, then $f$ is a soft loop in $U$ based at $a$ and $[\alpha \beta * \alpha] = [f]$. Hence $G$ is a strong SSLT space at $a$.

**Corollary 2.14:**
A soft topological group $G$ is an SSLT space if $G$ is an abelian group or a soft path connected space.

**Corollary 2.15:**
Let $G$ be a soft path connected topological group, then $\pi_{1}^{\text{wh}}(G, e_G)$ is a soft topological group.

**Conclusion:**
The study has reached that the soft quasitopological fundamental group of a soft connected and locally soft path connected space is a soft topological group.

**References**